The nonparaxial property of chirped pulsed beam

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Received April 3, 2003

The nonparaxial property of the chirped pulsed beam is analyzed both quantitatively and qualitatively. Through the qualitative investigation of the paraxial approximation condition, we show there are chirp-induced changes in the nonparaxial propagation of the chirped pulsed beam. A quantitative nonparaxial correction was developed by use of the perturbational technic and the Fourier transform for a few-cycle chirped pulsed beam with relative small chirp parameter. It was shown that the nonparaxial corrections were enhanced near the leading or trailing edge of pulse depending on weather the chirp parameter is positive or negative. An example for pulsed Gaussian beam driven by a chirped Gaussian pulse is shown in the numerical result to confirm our analysis.

OCIS codes: 320.1590, 320.2250, 320.5550.

The advent of the solid-state laser's capability of generating a train of ultrashort optical pulses has given rise to tremendous new phenomena related to spatiotemporal coupling in ultrashort pulsed light beams propagating in free space, linear , dispersive and nonlinear media[1−6]. It is well established that the propagation of ultrashort pulses cannot be assimilated to that of longer (quasi-monochromatic) pulses, and because the spatial and the temporal parts of the wave are not separable, the spatial evolution can affect the temporal behavior, and vice versa, even in free space. The spatiotemporal coupling giving rise to effects such as time-dependent diffraction patterns, the time-derivative effect, decrease of the optical period along the beam axis, pulse time delay, and red-shift toward the beam periphery etc., has been reported in Refs. [2,5,6].

The above-mentioned studies are all based on the standard paraxial theory, which is able to give an accurate description of the ultrashort pulsed beam propagation while the divergence angle is small and the beam width of the carrier frequency is much larger than its wavelength. However, there really exist ultrashort pulsed beams with large divergence angles or with the ultra-narrow waist at the center frequency. For those beams the paraxial theory will be invalid and it is necessary for us to make a nonparaxial correction to the paraxial pulsed beam solution. Though the nonparaxial propagation method of the monochromatic beam has been studied in detail[7−10], the nonparaxial property of the pulsed beam only has been studied in few articles such as Refs. [11,12], and many details have not yet been studied.

In practical applications, chirped pulses are used wildly. The relation between the nonparaxial property and the chirping property of pulsed beam was not illuminated yet. Because of the inherent spatiotemporal coupling of the pulsed beam, we could reasonably expect that the nonparaxial property of the pulsed beam should be influenced by its chirping property.

In this paper, based on the qualitative analysis of the paraxial approximation (PA) condition, we presented the chirp-induced changes in the nonparaxial propagation of the chirped pulsed beam. A quantitative nonparaxial correction was developed by use of the perturbational technic and the Fourier transform for a few-cycle chirped pulsed beam with relative small chirp parameter. It was shown that the nonparaxial corrections were enhanced near the leading or trailing edge of pulse depending on weather the chirp parameter is positive or negative. An example for pulsed Gaussian beam driven by a chirped Gaussian pulse is shown in the numerical result to confirm our analysis.

Let us firstly recall the basic wave equations of a linearly polarized pulsed beam $E(\vec{r}, z, t)$, where $\vec{r} = x\vec{e}_x + y\vec{e}_y$ is transverse coordinates, propagating along z direction in free space. The vectorial field is governed by

$$\left[\nabla^2 - \frac{1}{c^2}\partial_t^2\right] E = 0. \tag{1}$$

It is a well known fact that, outside the domain of PA, the electromagnetic field should be handled as a vectorial one since the coupling between the transverse components and the longitudinal one is not negligible. However, in free space, the longitudinal component can be evaluated from the knowledge of the transverse part[10,12], i.e. base on the relation $\nabla \cdot E = 0$. In this perspective and also for conciseness, we only consider the transverse components of the field in this paper.

Introducing the local variables $t' = t - z/c$ and $z' = z$, the transverse components are governed by

$$\left[\nabla_\perp^2 + \partial_{z'}^2 - \frac{2}{c}\partial_{z't'}^2\right] E = 0. \tag{2}$$

In general, it is inconvenient to obtain the analytic solution of Eq. (2). The PA is often introduced in dealing with the propagation of pulsed beam. And one can get the paraxial wave equation

$$\left[\nabla_\perp^2 - \frac{2}{c}\partial_{z't'}^2\right] E = 0, \tag{3}$$

under the PA condition[2]

$$|\partial_{z'} E| \ll \frac{1}{c}|\partial_{t'} E|. \tag{4}$$

Considering the pulsed beam whose pulse duration is longer than one optical oscillation period, it can be described by a carrier frequency ω_0 and an envelope Ψ, i.e.,

$E(t) = \Psi(\vec{r}, z', t')\exp(i\omega_0 t')$, as studied in Refs. [3−5]. Furthermore, when the pulse is a chirped one, it can be described as another form $E(t) = \Psi(\vec{r}, z', t')\exp(i\omega_0 t') = A(\vec{r}, z', t')\exp[i(\omega_0 t' + Ct'^2/\Delta t^2)]$, where $\Psi(\vec{r}, z', t') = A(\vec{r}, z', t')\exp(iCt'^2/\Delta t^2)$, T_0 is the oscillation period of the central frequency and C is the chirp parameter. The PA condition then can be expressed as

$$|\partial_{z'} A| \ll \frac{1}{c}|[i(\omega_0 + 2Ct'/\Delta t^2) + \partial_{t'}]A|. \tag{5}$$

Therefore we can see from Eq. (5), there are chirp-induced changes in nonparaxial property of the pulsed beam. Concretely, when the chirp parameter $C < 0$, there is a blue-shift near the leading edge whereas a red-shift near the trailing edge. As a result, the leading part of the pulse ($t' < 0$) satisfies the PA condition better than the trailing part($t' > 0$) does. On the contrary, the influence is just the reverse for chirp parameter $C > 0$.

In the following we derive a quantification relation to show the non-paraxial property of the chirped pulsed beam. It is convenient to deal with the linear propagation problem in frequency domain and the envelope spectrum $\widetilde{\Psi}(\vec{r}, z', \omega - \omega_0) = \int_{-\infty}^{\infty} \Psi(\vec{r}, z', t')e^{-(\omega-\omega_0)t'}dt'$ is governed by

$$[\nabla_\perp^2 + \partial_{z'}^2 - 2ik\partial_{z'}]\widetilde{\Psi} = 0. \tag{6}$$

And when the PA condition is satisfied, Eq. (6) deduces

$$[\nabla_\perp^2 - 2ik\partial_{z'}]\widetilde{\Psi}^{(p)} = 0. \tag{7}$$

The paraxial Eq. (7) has received extensive attention, and paraxial solution $\widetilde{\Psi}^{(p)}$ and its expression in the temporal domain $\Psi^{(p)}$ have been reported in Refs. [2,5,6].

When the beam width of the central frequency is as small as the wavelength, the paraxial solution requires a nonparaxial correction. Fortunately, the nonparaxial correction is of the order of $(ka)^{-2}\widetilde{\Psi}^{(p)}$, where a is the beam width of the central frequency. Therefore we can consider the nonparaxial correction as the perturbation and try a nonparaxial solution in the form of $\widetilde{\Psi} = \widetilde{\Psi}^{(p)} + \widetilde{\Psi}^{(c)}$, where $\widetilde{\Psi}^{(c)}$ is the nonparaxial correction, i.e., the perturbation term (here we only take the first order perturbation into account). $\widetilde{\Psi}^{(p)}$ and $\widetilde{\Psi}^{(c)}$ satisfy

$$[\nabla_\perp^2 - 2ik\partial_{z'}]\widetilde{\Psi}^{(p)} = 0,$$

$$[\nabla_\perp^2 - 2ik\partial_{z'}]\widetilde{\Psi}^{(c)} = -\partial_{z'}^2\widetilde{\Psi}^{(p)}. \tag{8}$$

It is straightforward calculation to verify that the non-paraxial correction in the frequency domain is given by

$$\widetilde{\Psi}^{(c)} = \frac{-iz'}{2k}\partial_{z'}^2\widetilde{\Psi}^{(p)}. \tag{9}$$

And we can use the inverse Fourier transform to get the non-paraxial correction in the temporal domain as

$$\Psi^{(c)} = \int \frac{-iz'}{2k}\partial_{z'}^2\widetilde{\Psi}^{(p)}e^{i(\omega-\omega_0)t}d\omega. \tag{10}$$

However, because of the mathematic complication as can be seen from the above equation, it is not convenient to get the analytical expression of the non-paraxial correction in the temporal domain directly from Eq. (10). It is noticed that the half-width of spectrum of the m-cycle chirped pulsed beam is approximative

$$\Delta\omega = \frac{\sqrt{1 + C^2}}{2m\pi}\omega_0. \tag{11}$$

When the pulse duration is correspondingly long and the chirp number C is relatively small, i.e. $|C| < m$, we have the relation $\Delta\omega \ll \omega_o$. So the integration domain in Eq. (10) is effectively limited to the small interval while $\widetilde{\Psi}^{(0)}$ takes significant values. It means that we can make an approximation $\frac{1}{k} \simeq \frac{1}{k_0}(1 - \frac{\omega-\omega_0}{\omega_0})$, and the nonparaxial correction is equivalent to

$$\Psi^{(c)} = \frac{-iz'}{2k_0}(1 + \frac{i}{\omega_0}\partial_{t'})\partial_{z'}^2\Psi^{(p)}. \tag{12}$$

According to Eq. (11), for pulsed beam with no chirping, Eq. (12) is appropriate to describe the few-cycle pulsed beam down to the limit of one cycle. And with the increase of the chirp parameter C, the pulse duration that Eq. (12) can describe will increase as well.

Now we take the pulsed Gaussian beam[6] driven by a chirped Gaussian pulse to test our analysis, i.e.

$$\Psi^{(p)}(\vec{r}, z', t') = \exp[-\frac{t_c'^2}{\Delta t^2}(1 + iC)]\frac{iz_R}{q(z)}\exp\left[\frac{-ik_0 r^2}{2q(z)}\right],$$

where $t_c' = t' - \frac{r^2}{2cq(z)}$ is the complex time. The beam width at central frequency is $a = \lambda_0 = 1$ μm. The on-axis ($\vec{r} = 0$) distributions of relative corrections $\Delta|\Psi| = (|\Psi^{(p)}|^2 - |\Psi|^2)/|\Psi^{(p)}(0, z', 0)|^2$ are given in Fig. 1 for pulse with different chirp parameters. The pulsed duration is $\Delta t = 3T_0$ whereas chirp parameters are $C = -3$, 0 and 3 to guarantee the condition $\Delta\omega \ll \omega_o$. When the chirp parameter $C = -3$, the nonparaxial correction near the leading edge of the pulse ($t' < 0$) is smaller than that near the trailing edge ($t' > 0$). On the contrary, if the chirp parameter $C = 3$, the influence of the chirp on the nonparaxial property is just the reverse. And we have investigated the Gaussian beam driven by other chirped pulses such as chirped Lorentz pulse,

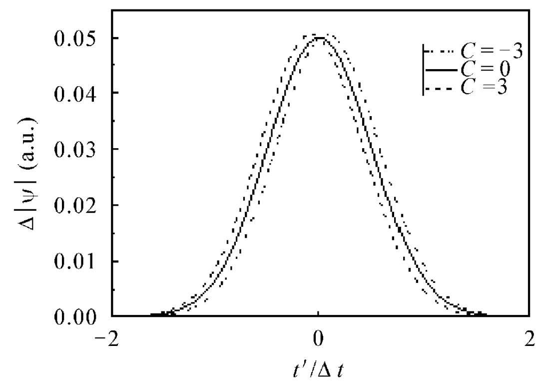

Fig. 1. The on-axis distributions of relative corrections $\Delta|\Psi| = (|\Psi^{(p)}|^2 - |\Psi|^2)/|\Psi^{(p)}(0, z', 0)|^2$ for pulse $\Delta t = 3T_0$ with different chirp parameters $C = -3, 0, 3$, respectively.

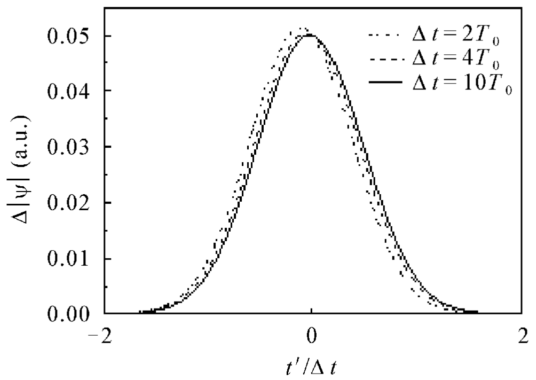

Fig. 2. The on-axis distributions of relative corrections $\Delta|\Psi| = (|\Psi^{(p)}|^2 - |\Psi|^2)/|\Psi^{(p)}(0,z',0)|^2$ for pulse $\Delta t = 2T_0, 4T_0, 10T_0$ with chirp parameters $C = 3$, respectively.

chirped Hyperbolic-Secant pulse and chirped Poisson-spectrum pulse *et al.*, they all have the same nonparaxial property as the chirped Gaussian one does. It shows that the numerical result consists with the above mentioned analysis.

In addition, the nonparaxial property of the chirped pulsed beam can also be influenced by the pulse duration, as shown in Fig. 2. The chirp parameter is a fixed one ($C = 3$) and pulsed duration increases from $2T_0$ to $10T_0$. As the pulse duration decreases, the asymmetric of the relative correction is strengthened, and the influence of the chirp on the nonparaxial propagation of the pulsed beam is strengthened as well, in that the spatiotemporal coupling in the nonparaxial correction arises with the shortening of the pulse duration and broadening of the spectrum width.

In conclusion, the chirp-induced changes in the nonparaxial propagation of the chirped pulsed beam were investigated both quantitatively and qualitatively in this paper. The nonparaxial property of the chirped pulsed beam is qualitatively analyzed by directly investigating the PA condition of the chirped pulsed beam. And a quantitative nonparaxial correction was developed by use of the perturbational technic and the Fourier transform for a few-cycles chirped pulsed beam with relative small chirp parameter. It was shown that the nonparaxial corrections were enhanced near the leading or trailing edge of pulse depending on weather the chirp parameter is positive or negative. An example for pulsed Gaussian beam driven by a chirped Gaussian pulse is shown in the numerical result to confirm our analysis.

Although our quantitative nonparaxial correction was obtained for a few-cycle chirped pulse with relative small chirp parameter $|C| < m$, the qualitative analysis of chirp-induced changes is still valid for large chirped pulse in general. The more quantitative investigation of chirp-induced changes for large chirp should be carried out in future. This particular property of the chirped pulsed beam can be used as a theoretic reference in the design of experiments with chirped pulsed beams.

This work was supported by the National Natural Science Foundation of China under (No. 60278013), the Team Project of Natural Science Foundation of Guangdong Province (No. 20003061), the Foundation of National High Technology Development Program of China, and the Fok Yin Tung High Education Foundation of the Ministry of Education of China (No. 81058). W. Hu is the author to whom the correspondence should be addressed, his e-mail address is huwei@scnu.edu.cn.

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