A scheme for the generation of two-mode atomic laser

Chunjia Huang (黄春佳), Ming Zhou (周 明), and Kewei Mo (莫克威)

Department of Physics and Electronic Science, Changsha University of Science & Technology, Changsha 410077

Received July 2, 2003

The quantum dynamic behavior of the system composed of V-type three-level atomic Bose-Einstein condensate (BEC) interacting with two-mode coherent light field has been studied. The results show that the atoms of V-type three-level atomic BEC, which are excited to higher-level states under the action of light field, still keep their properties of coherent states. It demonstrates theoretically that two-mode atomic laser may be prepared by V-type three-level atomic BEC.

OCIS codes: 140.1340, 270.3430, 020.5580.

In 1995, American physicists observed Bose-Einstein condensation of neutral alkaline metal atoms in laboratory[1,2]. In 1997, the group led by W. Ketterle in Massachusetts Institute of Technology, B. P. Anderson and others in Yale University manufactured atomic laser in succession[3,4]. These progresses of milestone opened new research field of physics. Experimental and theoretical studies on the generation of atomic Bose-Einstein condensate (BEC), its properties and its interaction with light field were carried out, and a series of important findings were achieved[5−13]. In this paper, we present a theoretical model for the interaction system of two-mode light field and V-type three-level atomic BEC and the quantum properties of atoms in untrapped states in the system of two-mode coherent light field and V-type three-level atomic BEC. The results show that the atoms of V-type three-level atomic BEC, which are excited to untrapped states under the action of light field, still keep their property of coherent states. Thus it demonstrates theoretically that two-mode atomic laser may be generated through interaction of V-type three-level atomic BEC with two-mode coherent light field.

The Hamiltonian of the interaction system composed of V-type three-level atomic BEC and two-mode light field, in rotating wave approximation can be expressed as

$$H = \omega_{01} b_1^+ b_1 + \omega_{02} b_2^+ b_2 + \omega_1 a_1^+ a_1 + \omega_2 a_2^+ a_2 + \varepsilon'(a_1 b_0 b_1^+ + a_1^+ b_0^+ b_1 + a_2 b_0 b_2^+ + a_2^+ b_0^+ b_2), \quad (1)$$

where b_i^+ and b_i denote creation and annihilation operators in ith ($i = 0, 1, 2$) atomic state, respectively, and a_i^+ and a_i denote creation and annihilation operators in ith ($i = 1, 2$) mode light field respectively, and ω_i denotes the circular frequency of ith ($i = 1, 2$) mode light field, and ω_{0i} denotes the transition frequency between atomic ground state and ith ($i = 1, 2$) excited state, and ε' denotes the coupling coefficient between atom and light field. For simplicity, we discuss the case of resonance, assuming $\omega_1 = \omega_{01}$ and $\omega_2 = \omega_{02}$.

Let all atoms be in ground state and Bose-Einstein condensation occur at initial moment, the excited state being vacuum. The state vector of system is expressed as

$$|\psi(0)\rangle = |\beta_0\rangle_0 \otimes |\Phi(0)\rangle, \quad (2)$$

where $|\beta_0\rangle_0$ is the eigenvector of annihilation operator b_0 characterizing that the atoms in ground state among which Bose-Einstein condensation occurs are in coherent state[6] and it holds $b_0|\beta_0\rangle_0 = \sqrt{N_0}e^{-i\theta}|\beta_0\rangle_0, N_0$ being the average number of atoms in state of $|\beta_0\rangle_0$. And $|\Phi(0)\rangle = |0\rangle_1 \otimes |0\rangle_2 \otimes |\alpha_1, \alpha_2\rangle_p$, where $|0\rangle_1$ and $|0\rangle_2$ represent that two excited states of atom are both vacuum at initial moment and $|\alpha_1, \alpha_2\rangle_p = |\alpha_1\rangle \otimes |\alpha_2\rangle$ is coherent state of two-mode light field, and $|\alpha_1, \alpha_2\rangle_p$ can be expressed as[14]

$$|\alpha_1, \alpha_2\rangle_p = D_1(\alpha_1) D_2(\alpha_2) |0\rangle_p, \quad (3)$$

where

$$D_1(\alpha_1) = \exp(\alpha_1 a_1^+ - \alpha_1^* a_1), \quad (4)$$

and

$$D_2(\alpha_2) = \exp(\alpha_2 a_2^+ - \alpha_2^* a_2). \quad (5)$$

They denote translation operators for two-mode light field of circular frequency ω_1 and ω_2, respectively.

For the convenience to solve the dynamic equation of the system, we discuss only the case of weak light field under Bogoliubov approximation[15], that is, assuming that the number of atoms in Bose-Einstein condensation state at initial moment is large enough, so as to neglect the slow variation of atom number of ground state in the interaction with light field and substitute b_0 and b_0^+ by $\sqrt{N_0}e^{-i\theta}$ and $\sqrt{N_0}e^{i\theta}$. Hence we got the matrix formulation of Heisenberg equation relating to a_1, a_2, b_1 and b_2 as

$$i\frac{\partial}{\partial t}\begin{pmatrix} b_1(t) \\ b_2(t) \\ a_1(t) \\ a_2(t) \end{pmatrix} = \begin{pmatrix} \omega_{01} & 0 & \varepsilon e^{-i\theta} & 0 \\ 0 & \omega_{02} & 0 & \varepsilon e^{-i\theta} \\ \varepsilon e^{i\theta} & 0 & \omega_1 & 0 \\ 0 & \varepsilon e^{i\theta} & 0 & \omega_2 \end{pmatrix} \begin{pmatrix} b_1(t) \\ b_2(t) \\ a_1(t) \\ a_2(t) \end{pmatrix}, \quad (6)$$

where $\varepsilon = \varepsilon'\sqrt{N_0}$. Solving Eq. (6), we obtain

$$a_1(t) = \alpha_{1a}(t) a_1(0) + \beta_{1a}(t) b_1(0), \quad (7)$$

$$a_2(t) = \alpha_{2a}(t) a_2(0) + \beta_{2a}(t) b_2(0), \quad (8)$$

$$b_1(t) = \alpha_{1b}(t) a_1(0) + \beta_{1b}(t) b_1(0), \quad (9)$$

$$b_2(t) = \alpha_{2b}(t) a_2(0) + \beta_{2b}(t) b_2(0), \quad (10)$$

where

$$\alpha_{1a}(t) = \beta_{1b}(t) = \cos(\varepsilon t)\mathrm{e}^{-i\omega_1 t}, \quad (11)$$

$$\beta_{1a}(t) = -i\sin(\varepsilon t)\mathrm{e}^{i\theta}\mathrm{e}^{-i\omega_1 t}, \quad (12)$$

$$\alpha_{1b}(t) = -i\sin(\varepsilon t)\mathrm{e}^{-i\theta}\mathrm{e}^{-i\omega_1 t}, \quad (13)$$

$$\alpha_{2a}(t) = \beta_{2b}(t) = \cos(\varepsilon t)\mathrm{e}^{-i\omega_2 t}, \quad (14)$$

$$\beta_{2a}(t) = -i\sin(\varepsilon t)\mathrm{e}^{i\theta}\mathrm{e}^{-i\omega_2 t}, \quad (15)$$

$$\alpha_{2b}(t) = -i\sin(\varepsilon t)\mathrm{e}^{i\theta}\mathrm{e}^{-i\omega_2 t}. \quad (16)$$

According to the hypothesis of approximation mentioned in section II, the slow variation of atom number of BEC in the process may be neglected and the system remains in coherent state $|\beta_0\rangle_0$ at any time, we express the evolution of system's state vector at any time as

$$|\psi(t)\rangle = |\beta_0\rangle_0 \otimes |\Phi(t)\rangle. \quad (17)$$

In normal case, $|\Phi(t)\rangle$ is an entanglement of atom states with photon states. But if a suitable initial state of light field has been chosen, $|\Phi(t)\rangle$ can be expressed as a direct product between the state vector of atom state and that of light field state, whereas the system's state vector possesses factorized structure.

For the case that the initial light field is of two-mode coherent state, we have

$$\begin{aligned}|\Phi(t)\rangle &= U(t)|\Phi(0)\rangle = U(t)|0\rangle_1 \otimes |0\rangle_2 \otimes |\alpha_1, \alpha_2\rangle_p \\ &= U(t)\exp[\alpha_1 a_1^+(0) - \alpha_1^* a_1(0) + \alpha_2 a_2^+(0) \\ &\quad -\alpha_2^* a_2(0)]U^+(t)U(t)|0\rangle_1 \otimes |0\rangle_2 \otimes |0\rangle_p. \quad (18)\end{aligned}$$

Using $c(t) = U^+(t)c(0)U(t)$, it is easy to get

$$\begin{aligned}|\Phi(t)\rangle &= \exp[\alpha_1 a_1^+(-t) - \alpha_1^* a_1(-t) + \alpha_2 a_2^+(-t) \\ &\quad -\alpha_2^* a_2(-t)]|0\rangle_1 \otimes |0\rangle_2 \otimes |0\rangle_p. \quad (19)\end{aligned}$$

Substitute Eqs. (7) – (10) into Eq. (19) and put the equation in order

$$\begin{aligned}|\Phi(t)\rangle &= |\alpha_1\beta_{1a}^*(-t)\rangle_1 \otimes |\alpha_2\beta_{2a}^*(-t)\rangle_2 \\ &\quad \otimes|\alpha_1\alpha_{1a}^*(-t), \alpha_2\alpha_{2a}^*(-t)\rangle_p. \quad (20)\end{aligned}$$

The total state vector of system is expressed as

$$|\psi(t)\rangle = |\beta_0\rangle_0 \otimes |\beta_1(t)\rangle_1 \otimes |\beta_2(t)\rangle_2 \otimes |\alpha_1'(t), \alpha_2'(t)\rangle, \quad (21)$$

where

$$|\beta_1(t)\rangle_1 = \exp[\beta_1(t)b_1^+(0) - \beta_1^*(t)b_1(0)]|0\rangle_1, \quad (22)$$

and

$$|\beta_2(t)\rangle_2 = \exp[\beta_2(t)b_2^+(0) - \beta_2^*(t)b_2(0)]|0\rangle_2. \quad (23)$$

In Eqs. (21) – (23), we have

$$\beta_0 = \sqrt{N_0}\mathrm{e}^{-i\theta}, \quad (24)$$

$$\beta_1(t) = \alpha_1\beta_{1a}^*(-t), \quad (25)$$

$$\beta_2(t) = \alpha_2\beta_{2a}^*(-t), \quad (26)$$

$$\alpha_1'(t) = \alpha_1\alpha_{1a}^*(-t), \quad (27)$$

$$\alpha_2'(t) = \alpha_2\alpha_{2a}^*(-t). \quad (28)$$

Equations (22) and (23) indicate that, after the atoms of ground state in trapped state among V-type three-level atomic BEC are excited to two untrapped state under the action of two-mode light field, they still have their characteristics of coherent state. And if they are output in suitable method, a coherent atomic beam containing two energy-components or two-mode atomic laser may be obtained.

We can calculate the atom number in untrapped state $|\beta_1(t)\rangle_1$ and $|\beta_2(t)\rangle_2$ at the moment t, respectively

$$N_{1b} = \overline{N}_{1a}\sin^2(\varepsilon t), \quad (29)$$

$$N_{2b} = \overline{N}_{2a}\sin^2(\varepsilon t), \quad (30)$$

where, $\overline{N}_{ia} = \langle\psi(0)|a_i^+ a_i|\psi(0)\rangle$ $(i = 1, 2)$, namely $\overline{N}_{ia}$ is the average photon number in initial state of system. These results show that the Rabi oscillating frequency of atom number in each untrapped state is proportional to the intensity of interaction between light field and atoms, and the atom number in each untrapped state is only dependent on the average photon number of initial light field at any moment.

In this paper, the quantum dynamic behavior of interaction system composed of V-type three-level atomic BEC and two-mode coherent light field has been studied. Adopting Bogoliubov approximation, a linear dynamic equation for the interaction system between light field and atomic BEC is obtained, system's time-dependent state vector of factorized structure is constructed and the possibility for preparing two-mode atomic laser using interaction between V-type three-level atomic BEC and two-mode coherent light field is analyzed. The results show that, after the atoms of ground state in trapped state are excited to two untrapped states under the action of two-mode light field, they still have their characteristics of coherent state. And if they are coupling-output in pulse, a coherent atomic beam containing two energy-components or two-mode atomic laser may be obtained. Hence it theoretically demonstrates that two-mode atomic laser may be generated through the interaction between V-type three-level atomic BEC and two-mode coherent light field.

C. Huang's e-mail address is huangcj@mail.csuep.edu.cn.

References

1. M. H. Anderson, J. R. Ensher, M. R. Matthews, C. E. Wieman, and E. A. Cornell, Science **269**, 198 (1995).
2. K. B. Davis, M. O. Mewes, M. R. Andrews, N. J. van Druten, D. S. Durfee, D. M. Kurn, and W. Ketterle,

Phys. Rev. Lett. **75,** 3969 (1995).
3. M. O. Mewes, M. R. Andrews, D. S. Durfee, D. M. Kurn, C. G. Townsend, and W. Ketterle, Phys. Rev. Lett. **78,** 582 (1997).
4. B. P. Anderson and M. A. Kasevich, Science **282,** 1686 (1998).
5. C. P. Sun, H. Zhan, Y. X. Miao, and J. M. Li, Commun. Theor. Phys. **29,** 161 (1998).
6. H. Jing, Y. A. Han, J. L. Chen, and Y. X. Miao, Chin. Phys. Lett. **17,** 862 (2000).
7. H. Jing, J. L. Chen, and M. L. Ge, Phys. Rev. A **63,** 15601 (2001).
8. H. Jing and M. L. Ge, Science in China A (in Chinese) **31,** 725 (2001).
9. L. M. Kuang and Z. Y. Zeng, Chin. Phys. Lett. **15,** 703 (1998).
10. H. Huang and S. Q. Li, Chin. Phys. Lett. **16,** 9 (1999).
11. J. S. Peng and G. X. Li, *Introduction of Modern Quantum Optics* (in Chinese) (Science in China Press, Beijing, 1996).
12. G. J. Ni and S. Q. Chen, *Advanced Quantum Mechanics* (in Chinese) (Fudan University Press, Shanghai, 2000).